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A simple calculation method for the self- and mutual-radiation impedance of flexible rectangular patches in a rigid infinite baffle

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Abstract

A numerical model has been developed to calculate the self- and mutual-radiation impedance in the cases of uniformly and flexibly vibrating rectangular patches in a rigid infinite baffle. The spatial convolution approach is employed here to derive general expressions for the radiation impedance of a rectangular radiator in the form of simple integrals, which allows a fast evaluation numerically. The presented integral solution agrees with that obtained for the mutual-radiation impedance of a uniformly vibrating rectangular piston by the use of the classical approach. The numerical results of self-radiation impedance of a square piston are compared with the tabulated values published previously. As examples of flexibly vibrating rectangular patch, a closed-form expression is first given for the radiation impedance in the normal mode of vibration. The numerical results reveal that the computation time in obtaining accurate calculations is greatly reduced by using the proposed method.

1. Introduction

The calculation of the radiation impedance is fundamental to evaluate the acoustic characteristics. It is often necessary to take into account the mutual impedance effects in noise

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control and the design of transducer arrays. Radiation impedance computation has received enormous attention in the literature because of its importance in the analysis of a variety of acoustical systems [1,2]. Previous investigators have studied self-radiation impedance for particular baffles [3–5], while also considering the mutual impedance for various radiators such as sources on a sphere [6], circular and rectangular pistons on an infinite plane [6–9], pistons on a cylinder, etc. [10].

Rectangular radiator has practical interest because it is usually used in the transducer array. In 1964, Arase [8] proposed a classic approach to calculate the radiation impedance of a rectangular piston. The two identical rectangular pistons are assumed in same plane and have parallel sides. An integral solution was given to simplify the evaluation for the mutual impedance. In 1971, Stepanishen [11] suggested a time domain solution of the radiation impedance of a square piston. The calculation formula for square piston is a closed form. Later, Burnett and Soroka [12] introduced another kind of single integral, which contains a highly oscillatory integral kernel for calculating the self-radiation impedance. They used a highly efficient numerical algorithm to obtain the results. However, expressions of the self- and mutual-radiation impedance for rectangular pistons are very complicated and hard to obtain an analytical solution. The evaluation of the mutual impedance is rather cumbersome. The standard quadrature computation algorithms, such as Gaussian, Simpson, Newton–Coters, Romberg, Monte Carlo, etc., are too inefficient for calculating the numerical results. This is because of their high-dimensional integral. Furthermore, the integral kernel contains numerical singularity points when self-radiation impedance is calculated.

Recently, emphasis has been given to the numerical method. Bank and Wright [13] suggested quadruple integral equations by using geometric relations among rectangular pistons, and presented the tabulated data for various aspect ratios. Lee and Seo [14] employed the similar method to estimate radiation power for the planar array acoustic transducer considering the mutual coupling effect [15]. In addition, a modal Pritchard approximation has been developed to compute the mutual impedance for acoustically hard arrays [16], whereas all those formulations presented are based on uniformly vibrating rectangular patches, which may not be extended to find acoustic interaction for a flexibly vibrating case. Generally, to calculate the mutual- and self-radiation impedance of flexible rectangular patches, evaluating a quadruple integral is needed. Li and Gibeling [17] investigated the calculation of mutual-radiation resistance from cross-model coupling for a simply supported rectangular plate and their effects on the radiated sound power. It was shown that, by recasting the quadruple integrals into several double integrals, the mutual-radiation resistance could be obtained easily in the whole frequency range. More recently, in their compressive paper, Pierce et al. [18] have discussed previous work and simplified the numerical elevation of the radiation impedance for a single rectangular aperture when the basis functions are expressible as a sum of products of exponential functions. The radiation impedance matrix has been studied in a systematical manner and can be applied in many acoustic problems. Although various methods have been presented for simplifying the numerical integration for a rectangular radiator, there are few publications on the characteristics of the mutual-radiation impedance resulting from two flexible rectangular patches. Thus, it would be appropriate to arrive at a solution for the mutual-radiation impedance of two flexible rectangular patches in a rigid infinite baffle.

In this paper, an efficient calculation method has been developed, which is suited to the determination of the radiation impedance in the cases of uniformly and flexibly vibrating rectangular patches. By using the spatial convolution approach, the general solution can be expressed as a double integral. From some examples, the present algorithm provides numerical results for the self- and mutual-radiation impedance, which are in good agreement with those obtained directly by the classical approach. The comparison demonstrates that the computational time of calculating the radiation impedance can be dramatically reduced.

2. Theory

Consider two planar radiators in an infinite rigid plane with velocity distribution of the form $v_1(x,y) = U_1 u_1(x,y)$ on one patch and $v_2(x,y) = U_2 u_2(x,y)$ on the other patch. U_1 and U_2 denote the reference velocities on patch #1 and patch #2, respectively. The reference velocity is referred to be the average normal surface velocity. If the average normal surface velocity is zero, or very small, the velocity at the center should be referred to the reference velocity [9]. $u_1(x,y)$ and $u_2(x,y)$ denote the velocity distribution functions on patch #1 and patch #2, respectively.

For a given vibration velocity $v_1(x',y')$, the sound pressure is given by the following formula:

$$p_1(x,y) = \frac{\mathrm{i}ck\rho}{2\pi} \int_{S_1}\int v_1(x',y') \frac{\mathrm{e}^{\mathrm{i}kR}}{R}\,\mathrm{d}s', \tag{1}$$

where S_1 denotes the surface area of the radiator, c is the velocity of sound in the medium, ρ is the density of the medium, k is the wave number and $R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$ is the distance between the source point and field point.

Mutual-radiation impedance of the radiators Z_{12} is defined as the force f_{12} on the radiator (patch #2) resulting from the vibration of the other radiator (patch #1), divided by the reference velocity U_1 of that vibrating radiator:

$$Z_{12} = \frac{f_{12}}{U_1}. \tag{2}$$

The force on patch #2 due to the vibration of patch #1 is given as

$$f_{12} = \int_{S_2}\int p_1(x,y) u_2(x,y)\,\mathrm{d}s. \tag{3}$$

Then, the mutual-radiation impedance can be obtained as

$$Z_{12} = \frac{\mathrm{i}\rho ck}{2\pi} \int_{S_1}\int \int_{S_2}\int u_1(x',y') u_2(x,y) \frac{\mathrm{e}^{-\mathrm{i}kR}}{R}\,\mathrm{d}s'\,\mathrm{d}s, \tag{4}$$

where S_2 denotes the surface area of transducer #2. It can be seen that the principle of reciprocation is satisfied by this definition. That means $Z_{12} = Z_{21}$. In the following discussion, we will use Z_m to denote mutual-radiation impedance.

As two identical planar sources coincide with each other, Eq. (4) becomes the expression of self-radiation impedance,

$$Z_s = \frac{\mathrm{i}\rho ck}{2\pi} \int_{S_1}\int \int_{S_1}\int u_1(x',y') u_1(x,y) \frac{\mathrm{e}^{-\mathrm{i}kR_s}}{R_s}\,\mathrm{d}s'\,\mathrm{d}s, \tag{5}$$

where $R_s = \sqrt{(x-x')^2 + (y-y')^2}$ is the distance between the elements ds and ds'. Obviously, Eqs. (4) and (5) are all quadruple integrals, which make difficult numerical computation.

Fig. 1 shows two coplanar rectangular patches, where w_x and w_y are referred to the width of the rectangular patch in x and y directions, respectively. The radiation impedance expressions will be simplified to double integrals for easy evaluation.

From Eq. (4), the mutual-radiation impedance is given by

$$Z_m = \frac{\mathrm{i}\rho ck}{2\pi}\int_{-w_x/2}^{w_x/2}\int_{-w_y/2}^{w_y/2}\int_{-w_x/2}^{w_x/2}\int_{-w_y/2}^{w_y/2} u_1(x',y')u_2(x,y)\frac{\mathrm{e}^{-\mathrm{i}kR_m}}{R_m}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y', \tag{6}$$

where $R_m = \sqrt{(x_2 - x_1 + x - x')^2 + (y_2 - y_1 + y - y')^2 + (z_2 - z_1)^2}$. The point (x_1, y_1, z_1) and (x_2, y_2, z_2) are referred to the center points of patch #1 and patch #2, respectively. Similarly, as two rectangular patches coincide, i.e., $x_1 = x_2$, $y_1 = y_2$, and $z_1 = z_2$, Eq. (6) becomes the expression of self-radiation impedance.

Using the Dirac delta function, the Green's function in Eq. (6) can be written as

$$\frac{\mathrm{e}^{-\mathrm{i}kR_m}}{R_m} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(u-(x-x'))\delta(v-(y-y'))\frac{\mathrm{e}^{-\mathrm{i}kr}}{r}\,\mathrm{d}u\,\mathrm{d}v, \tag{7}$$

where $r = \sqrt{(u + x_2 - x_1)^2 + (v + y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Substituting Eq. (7) into Eq. (6) and changing the integral order, we have the mutual impedance

$$Z_m = \frac{\mathrm{i}\rho ck}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s(u,v)\frac{\mathrm{e}^{-\mathrm{i}kr}}{r}\,\mathrm{d}u\,\mathrm{d}v, \tag{8}$$

where

$$s(u,v) = \int_{-w_x/2}^{w_x/2}\int_{-w_y/2}^{w_y/2}\int_{-w_x/2}^{w_x/2}\int_{-w_y/2}^{w_y/2} u_1(x',y')u_2(x,y)\delta(u-(x-x'))\delta(v-(y-y'))\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y'. \tag{9}$$

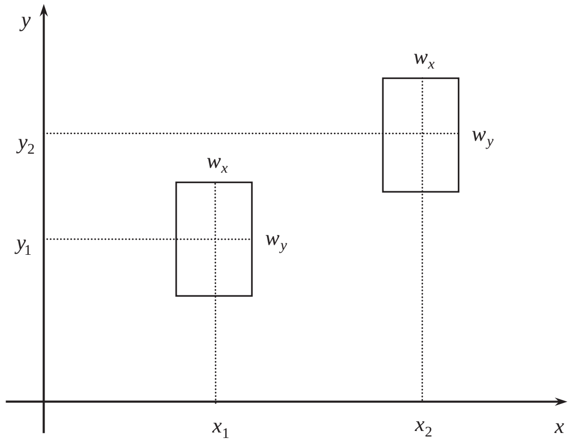


Fig. 1. The geometry of two rectangular patches.

After integrating over x and y, the spatial convolution formula is achieved as

$$s(u,v)=\int_{-w_x/2}^{w_x/2}\int_{-w_y/2}^{w_y/2}u_1(x',y')u_2(u+x',v+y')f_1(x',u)f_2(y',v)\,\mathrm{d}x'\,\mathrm{d}y', \tag{10}$$

where

$$f_1(x',u)=\begin{cases}1, & -\dfrac{w_x}{2}\leqslant u+x'\leqslant\dfrac{w_x}{2},\\ 0, & \text{other}\end{cases} \tag{10a}$$

and

$$f_2(y',v)=\begin{cases}1, & -\dfrac{w_y}{2}\leqslant v+y'\leqslant\dfrac{w_y}{2},\\ 0, & \text{other.}\end{cases} \tag{10b}$$

Because $-w_x/2\leqslant x'\leqslant w_x/2$ and $-w_y/2\leqslant y'\leqslant w_y/2$, from Eqs. (10a) and (10b), it can be derived that $-w_x\leqslant u\leqslant w_x$ and $-w_y\leqslant v\leqslant w_y$. Then Eq. (8) can be rewritten as

$$Z_m=\frac{\mathrm{i}\rho ck}{2\pi}\int_{-w_x}^{w_x}\int_{-w_y}^{w_y}s(u,v)\frac{\mathrm{e}^{-\mathrm{i}kr}}{r}\,\mathrm{d}u\,\mathrm{d}v. \tag{11}$$

Given the velocity distribution functions $u_1(x,y)$ and $u_2(x,y)$, the function $s(u,v)$ in Eq. (8) can be obtained by using Eq. (10). It should be noted that expression of $s(u,v)$ is usually in closed form for a normal velocity distribution. To illustrate in some detail the computations involved, we derive the function $s(u,v)$ in the next section.

3. The closed-form expression of function $s(u,v)$

In general, any arbitrary velocity distribution on a baffled rectangular patch may be represented by a series expansion if the set of basis functions is taken to satisfy different boundary conditions. A natural choice of basis functions would be products of trigonometric functions such as a sine–sine series or cosine–cosine series. In this paper, the origin of our coordinate system is assumed to be at the center of the patch; the distribution of velocity on a radiator can take the form $\cos(k_m^x x)\cos(k_n^y y)\,k_m^x=m\pi/w_x$ is the wave number of the mth basis function in the x-direction and $k_n^y=n\pi/w_y$ the wave number of nth basis function in the y-direction. It is clear that Eq. (10) can be evaluated analytically; so the initial fourfold integral for radiation impedance is reduced to a twofold integral.

3.1. Rigid piston

The simplest case is a baffled rigid-piston radiator, which is treated in many textbooks and some papers, but we include it to illustrate the method and for comparison. Consider two uniformly vibrating rectangular patches in a rigid infinite baffle, the velocity distribution functions can take the forms $u_1(x,y)=1$ and $u_2(x,y)=1$. It is the special case of cosine–cosine series $u(x,y)=\cos(m\pi x/w_x)\cos(n\pi y/w_y)$ as $m=n=0$.

From Eq. (10), we have,

$$s(u,v) = (w_x - |u|)(w_y - |v|). \tag{12}$$

After the above closed-form solution is substituted into Eq. (11), some analytical simplification of the mutual radiation impedance is obtained as

$$Z_m = \frac{\mathrm{i}\rho ck}{2\pi}\int_{-w_x}^{w_x}\int_{-w_y}^{w_y}(w_x - |u|)(w_y - |v|)\frac{\mathrm{e}^{-\mathrm{i}kr}}{r}\,\mathrm{d}u\,\mathrm{d}v. \tag{13}$$

This expression is the same as the intermediary result introduced by Arase [8]. Through algebraic deduction, Eq. (13) can be further reduced to one-dimensional integral.

3.2. Simply supported radiator

For the simply supported radiator, its velocity has maximum at the center of the patch and zero at the edge. One could choose the velocity distribution functions of two baffled rectangular patches in the forms

$$u_1(x,y) = \cos\left(\frac{m\pi}{w_x}x\right)\cos\left(\frac{n\pi}{w_y}y\right), \tag{14}$$

$$u_2(x,y) = \cos\left(\frac{k\pi}{w_x}x\right)\cos\left(\frac{l\pi}{w_y}y\right), \tag{15}$$

where m, n and k, l are odd integers denoting the number of antinode lines.

From Eq. (10), we have

$$s(u,v) = s_x(u)s_y(v), \tag{16}$$

where

$$s_x(u) = \frac{w_x - |u|}{2}\{\cos(u_1)\,\sin c(u_2) + \cos(u_3)\,\sin c(u_4)\}, \tag{16a}$$

$$s_y(v) = \frac{w_y - |v|}{2}\{\cos(v_1)\,\sin c(v_2) + \cos(v_3)\,\sin c(v_4)\}. \tag{16b}$$

In Eqs. (16a) and (16b), the coefficients u_i and v_i ($i = 1, 2, 3, 4$) are denoted as

$$u_1 = (m-k)\pi u/2w_x, \quad u_2 = (m+k)(w_x - |u|)\pi/2w_x, \quad u_3 = (m+k)\pi u/2w_x,$$
$$u_4 = (m-k)(w_x - |u|)\pi/2w_x, \quad v_1 = (n-l)\pi v/2w_y, \quad v_2 = (n+l)(w_y - |v|)\pi/2w_y,$$
$$v_3 = (n+l)\pi v/2w_y \quad \text{and} \quad v_4 = (n-l)(w_y - |v|)\pi/2w_y.$$

In addition, $\sin c(x) = \sin(x)/x$. Finally, the radiation impedance is derived as a two-dimensional integral equation

$$Z_m = \frac{\mathrm{i}\rho ck}{2\pi}\int_{-w_x}^{w_x}\int_{-w_y}^{w_y} s_x(u)s_y(v)\frac{\mathrm{e}^{-\mathrm{i}kr}}{r}\,\mathrm{d}u\,\mathrm{d}v, \tag{17}$$

which deduces to Eq. (13) as m, n, k and l are all zeros. It is worthy of note, if $m = k$, $n = l$ and two rectangular patches coincide, i.e., $x_1 = x_2$, $y_1 = y_2$, and $z_1 = z_2$, Eq. (17) becomes the expression of self-radiation impedance for a single flexible rectangular patch, which is similar to the analytical equation (27) derived in Ref. [18].

3.3. Clamped radiator

In this case, there is no exact solution of the fourth-order plate equation for a clamped boundary. We can make good use of the products of Warburton function [19] for a close approximation. Thus, the velocity distribution function of a baffled rectangular patch can be written as $u(x,y) = v_x(x)v_y(y)$, and

$$v_x(x) = \cos\left(\frac{\gamma_p x}{w_x}\right) + \frac{\sin(\gamma_p/2)}{\sinh(\gamma_p/2)}\cosh\left(\frac{\gamma_p x}{w_x}\right), \tag{18}$$

where γ_p is a root of the characteristic equation

$$\tan(\gamma_p/2) + \tanh(\gamma_p/2) = 0, \quad p = 2,4,6,\ldots, \tag{19}$$

where p indicates the number of nodal lines. Similarly, we can get the expression for $v_y(y)$ in y-coordinate.

To evaluate Eq. (10) in closed form, the even function $v_x(x)$ can be represented in the form of Fourier series expansion,

$$v_x(x) = \frac{a_0}{2} + \sum_{q=1}^{\infty} a_q \cos\left(\frac{2q\pi}{w_x}x\right), \tag{20}$$

where the Fourier coefficients

$$a_q = \frac{2}{w_x}\int_{-w_x/2}^{w_x/2} v_x(x)\cos\left(\frac{2q\pi}{w_x}x\right)\mathrm{d}x, \quad q = 0,1,2,\ldots.$$

Inserting Eq. (18) and integrating it yields

$$a_q = \frac{1}{1+\delta_q}\frac{(-1)^q 8\gamma_p^3 \sin(\gamma_p/2)}{\gamma_p^4 - 16q^4\pi^4}, \tag{21}$$

where $\delta_q = 1$ $(q = 0)$ or 0 $(q \neq 0)$. From Eq. (10), we have

$$s(u,v) = s_x(u)s_y(v), \tag{22}$$

where

$$
\begin{aligned}
s_x(u) &= \int_{-w_x/2}^{w_x/2} v_{x1}(x) v_{x2}(u+x) f_1(x,u)\,\mathrm{d}x \\
&= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_m a_n \int_{-w_x/2}^{w_x/2} \cos\left(\frac{2m\pi}{w_x}\right)\cos\left(\frac{2n\pi}{w_x}\right) f_1(x,u)\,\mathrm{d}x \\
&= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_m a_n f_x(m,n,u),
\end{aligned} \tag{23}
$$

where the coefficients a_m and a_n take the expression given in Eq. (21), but with δ_m and δ_n, respectively. The quantity $f_x(m,n,u)$ is the integral that appears in Eq. (23), which can be evaluated analytically as

$$
f_x(m,n,u) = \frac{w_x - |u|}{2}\{\cos(u_1)\sin c(u_2) + \cos(u_3) + \sin c(u_4)\}, \tag{24}
$$

where $u_1 = (m-k)\pi u/w_x$, $u_2 = (m+k)(w_x - |u|)\pi/w_x$, $u_3 = (m+k)\pi u/w_x$, and $u_4 = (m-k)(w_x - |u|)\pi/w_x$.

Similarly, the function $s_y(v)$ can be obtained as

$$
s_y(v) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} a_k a_l f_y(k,l,v), \tag{25}
$$

where the coefficients a_k and a_l take the expression given in Eq. (21) but with δ_k and δ_l, respectively, and the quantity $f_y(k,l,u)$ is given by

$$
f_y(k,l,v) = \frac{w_y - |v|}{2}\{\cos(v_1)\sin c(v_2) + \cos(v_3) + \sin c(v_4)\}, \tag{26}
$$

where $v_1 = (n-l)\pi v/w_y$, $v_2 = (n+l)(w_y - |v|)\pi/w_y$, $v_3 = (n+l)\pi v/w_y$ and $v_4 = (n-l)(w_y - |v|)\pi/w_y$. Thus, the closed-form solution to function $s(u,v)$ is

$$
s(u,v) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} a_m a_n a_k a_l f_x(m,n,u) f_y(k,l,v). \tag{27}
$$

Note that a truncated Fourier series is used to approximate the function $s(u,v)$ for simplification of the computation given in the next section.

4. Numerical results

The above equations (6) and (11) can be used to calculate the radiation impedances of uniformly and flexibly vibrating rectangular patches. Let $Z_m = \rho c A(R + \mathrm{i}X)$. Here R and X represent normalized radiation resistance and reactance, respectively; A denotes the area of the patch. All evaluations were done on a PC computer (Pentium IV 1.9 GHz) and the program codes were developed using FORTRAN 90 language. We will test the efficiency and accuracy of the proposed method through several examples.

Firstly, the numerical results obtained from Eq. (13) for the rectangular piston are compared with the tabulated results given by Burnett and Soroka [12]. The results in Table 1 clearly show that our results agree with the Burnett's results very well for various values of ka, where $a = \sqrt{A}$. The percentage differences between Burnett's results and ours are considerably less than 0.05%. In contrast, the computation errors derived from the quadruple surface integral method increase significantly as ka becomes larger [13].

Secondly, the radiation impedance of the flexible rectangle patch is evaluated for distributions of velocity on the radiator other than uniform, which may in some applications be more realistic. In the case of the simply-supported radiators, Eqs. (16), (17) and the quadruple integral (4) are used, respectively. For simplicity, the variables $m = k = 1$ and $n = l = 0$ are assumed for calculation. It implies that the velocity distribution is sinusoid in the x-direction with two nodal lines and uniform in the y-direction. To solve Eq. (4), the multilayer Gaussian quadrature algorithm is employed. We use ten Gaussian–Legendre nodes and weights, and split the intervals of the integral to get the desired numerical precision. In the case of clamped radiators, the velocity distribution function takes the form of Eq. (18). Eqs. (20)–(27) and Eq. (11) are employed. Similarly, the number of nodal lines in the x-direction is assumed to be $p = 2$. It corresponds to two baffled square patches clamped at $x = -w_x/2$ and $x = w_x/2$ with a rigid boundary in the y-direction. Tables 2 and 3 show the numerical results and the evaluation time consumed by using twofold integral and fourfold integral, respectively, for simply supported and clamped radiators. Obviously, the proposed

Table 1
Normalized self-radiation resistance and reactance of square piston

ka	Our results		Burnett's results [12]		Differences	
	R	X	R	X	$\Delta R/R$ (%)	$\Delta X/X$ (%)
0.1	0.00159067	0.04727767	0.0015907	0.0472786	0.0022	0.0020
0.2	0.00635207	0.09430686	0.0063521	0.0943088	0.0005	0.0021
0.3	0.01425251	0.14084102	0.0142525	0.1408439	−0.0001	0.0020
0.4	0.02523946	0.18663737	0.0252395	0.1866412	0.0002	0.0021
0.5	0.03924001	0.23145882	0.0392400	0.2314636	0.0000	0.0021
0.6	0.05616148	0.27507572	0.0561615	0.2750815	0.0000	0.0021
0.7	0.07589220	0.31726765	0.0758922	0.3172743	0.0000	0.0021
0.8	0.09830245	0.35782502	0.0983024	0.3578327	−0.0001	0.0021
0.9	0.12324558	0.39655066	0.1232456	0.3965593	0.0000	0.0022
1	0.15055921	0.43326126	0.1505592	0.4332708	0.0000	0.0022
2	0.51011721	0.66131165	0.5101172	0.6613308	0.0000	0.0029
3	0.87306171	0.61992498	0.8730617	0.6199537	0.0000	0.0046
4	1.07097152	0.41240951	1.0709715	0.4124478	0.0000	0.0093
5	1.07697326	0.21036721	1.0769733	0.2104150	0.0000	0.0227
6	0.98870257	0.12317300	0.9887026	0.1232262	0.0000	0.0432
7	0.92521441	0.14260903	0.9252144	0.1426650	0.0000	0.0392
8	0.93637777	0.18836267	0.9363778	0.1884227	0.0000	0.0319
9	0.99123608	0.19315527	0.9912361	0.1932208	0.0000	0.0339
10	1.03153947	0.15019029	1.0315395	0.1502609	0.0000	0.0470

Table 2
Normalized self-radiation resistance and reactance for a square simply supported radiator ($x_1 = x_2$, $y_1 = y_2$, and $z_1 = z_2$)

ka	Double integral			Quadruple integral		
	R	*X*	Time (s)	*R*	*X*	Time (s)
1.00	0.061757	0.204803	57.95	0.061757	0.204825	69436.27
5.00	0.582937	0.186536	58.15	0.582937	0.186647	69376.52
10.00	0.531951	0.046463	58.16	0.531951	0.046685	69512.35

Table 3
Normalized self-radiation resistance and reactance for a square clamped radiator ($x_1 = x_2$, $y_1 = y_2$ and $z_1 = z_2$)

ka	Double integral			Quadruple integral		
	R	*X*	Time (s)	*R*	*X*	Time (s)
1.00	0.244639	0.717296	66.80	0.244639	0.717359	70321.67
5.00	1.843234	0.369401	78.51	1.843236	0.369635	89517.97
10.00	1.687086	0.205079	85.22	1.687088	0.205547	89524.68

method has significantly high efficiency. Compared to the quadruple integral method, the computing time is greatly reduced. The ratio of the computing time reduction is more than 1000 times.

Finally, the self-radiation impedance and mutual-radiation impedance for flexible patches with different velocity distribution modes are evaluated. In most cases of interest, a complete set of basic functions can be represented as a sum of terms like products of Eqs. (14) and (15). The variables m, n, k and l are integers, which imply that the radiator is not only considered for a simply supported case. Table 4 shows the numerical results of the normalized self-radiation impedance of a square radiator with different distribution modes by using the proposed method. It is interesting to see that for some velocity distribution, the self-radiation resistance is smaller than others. It indicates that radiation power at this velocity distribution is less than the others. We also noted that the self-radiation resistance of a uniform vibration patch ($m = k = 0,\ n = l = 0$) is the largest one among all other modes. It means that a piston has the highest radiation efficiency. It should be mentioned that the sums of computing time of the data in Table 4 are less than one-point calculation time of the quadruple integral.

Figs. 2–4 give the normalized mutual radiation resistance and reactance for $ka = 0.1,\ 0.5$ and 1 as functions of the normalized separation distance when the square patches are lined up. The normalized center-to-center distance between two patches (d/a) is zero in the y-direction and it changes from 1 to 50 in the x-direction. One can see that the zeroes of different modes are at same points on the d/a-axis. The normalized mutual-radiation resistance and reactance are oscillatory decay as a function of d/a. As ka becomes larger, the normalized mutual-radiation impedance oscillates faster. This kind of oscillation is independent of the velocity distribution modes. For

Table 4
Normalized self-radiation resistance and reactance of a flexible square patch with different velocity distribution modes ($k = m,\ l = n$)

m	n	$ka = 0.1$		$ka = 1$		$ka = 10$	
		R	X	R	X	R	X
0	0	0.00159067	0.04727650	0.15055921	0.43324962	1.03153948	0.15011322
0	1	0.00064475	0.02189933	0.06175746	0.20479820	0.53195121	0.04644564
0	2	0.00000000	0.00638991	0.00001972	0.06488497	0.66599071	0.21373722
0	3	0.00007160	0.00672003	0.00645908	0.06524084	0.67563581	0.46934326
0	4	0.00000000	0.00357818	0.00000120	0.03591298	0.12039419	0.59146595
0	5	0.00002577	0.00382414	0.00231467	0.03748753	0.04101166	0.40844951
1	1	0.00032951	0.01121528	0.03154372	0.10486781	0.29358978	0.01894459
1	2	0.00003582	0.00398713	0.00348391	0.03952710	0.33522131	0.14542275
1	3	0.00001451	0.00324553	0.00136317	0.03237925	0.32040193	0.21731005
1	4	0.00000143	0.00178518	0.00006687	0.01774057	0.09055892	0.28093284
1	5	0.00000522	0.00180334	0.00047872	0.01792812	0.02823093	0.23308872
2	2	0.00019883	0.00647032	0.01878074	0.05967012	0.10116984	0.18590621
2	3	0.00011607	0.00542826	0.01127740	0.05214969	0.16727571	0.11501175
2	4	0.00000003	0.00265476	0.00028907	0.02743063	0.13968432	0.07805599
2	5	0.00001826	0.00227739	0.00164903	0.02229011	0.18346620	0.17336025
3	3	0.00020045	0.00686633	0.01901033	0.06379302	0.17538384	0.12371834
3	4	0.00005924	0.00408282	0.00597795	0.04034745	0.12163883	0.07259778
3	5	0.00000059	0.00240842	0.00014370	0.02451100	0.18341708	0.12138249
4	4	0.00019883	0.00628496	0.01881009	0.05789118	0.12552331	0.05713162
4	5	0.00009952	0.00475208	0.00973353	0.04584828	0.12906183	0.06833557
5	5	0.00019904	0.00641822	0.01885652	0.05927743	0.13451986	0.07030788

same size of the radiator, the higher the velocity distribution mode is, the smaller the radiation impedance value becomes.

Figs. 5 and 6 give the normalized mutual-radiation resistance and reactance for different velocity distribution modes as a function of separation distance (kd) between two patches. The normalized mutual radiation resistance and reactance also are oscillating decay functions along the kd-axis. It is obvious that for the same velocity distribution mode, different size of the patch has the same oscillatory frequency and the larger one has higher radiation impedance values.

5. Conclusion

A simplified method for calculating the radiation impedance of rectangular patches in a rigid infinite baffle was presented. The general solution for the self- and mutual-radiation impedance in the cases of uniform and nonuniform vibration was derived. As an example, a formula has been given to calculate the self- and mutual-radiation impedance of the rectangle patch with different velocity distribution by using a double integral, instead of quadruple integrals. Numerical results

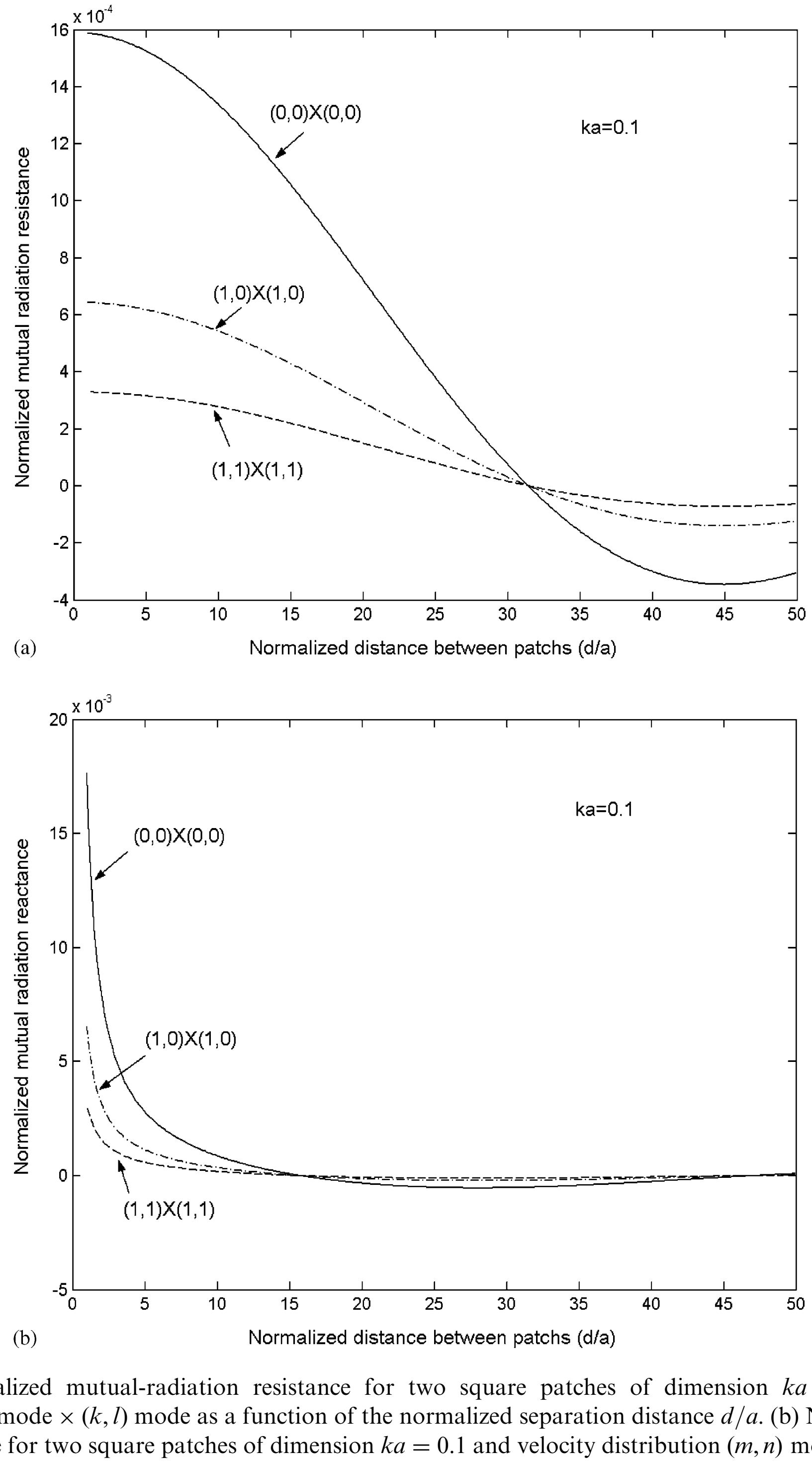
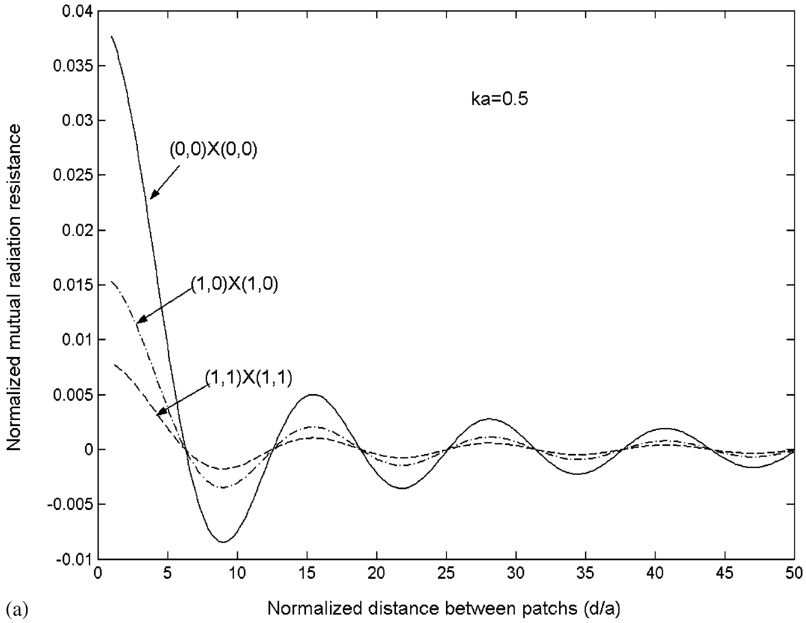
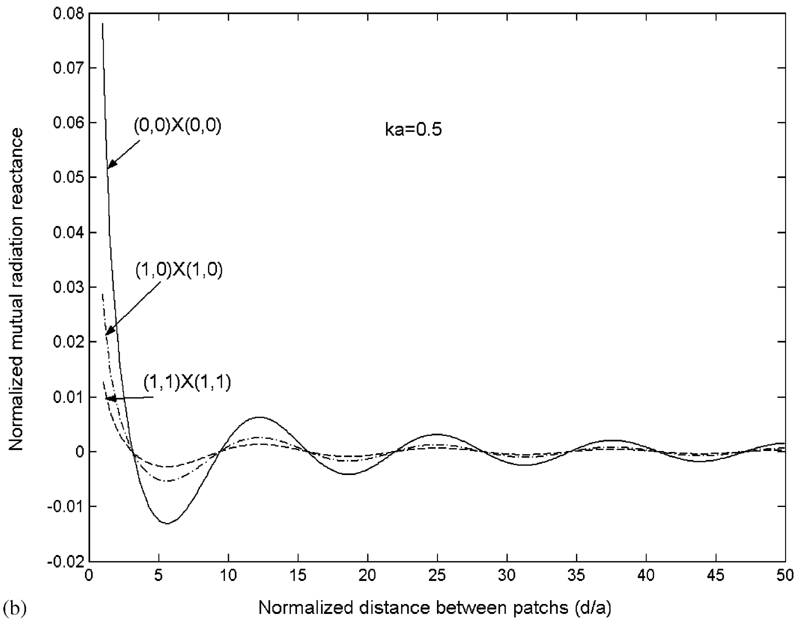


Fig. 2. (a) Normalized mutual-radiation resistance for two square patches of dimension $ka = 0.1$ and velocity distribution (m, n) mode $\times$ (k, l) mode as a function of the normalized separation distance d/a. (b) Normalized mutual-radiation reactance for two square patches of dimension $ka = 0.1$ and velocity distribution (m, n) mode $\times$ (k, l) mode as a function of the normalized separation distance d/a.



(a)



(b)

Fig. 3. (a) Normalized mutual-radiation resistance for two square patches of dimension $ka = 0.5$ and velocity distribution (m, n) mode $\times$ (k, l) mode as a function of the normalized separation distance d/a. (b) Normalized mutual-radiation reactance for two square patches of dimension $ka = 0.5$ and velocity distribution (m, n) mode $\times$ (k, l) mode as a function of the normalized separation distance d/a.

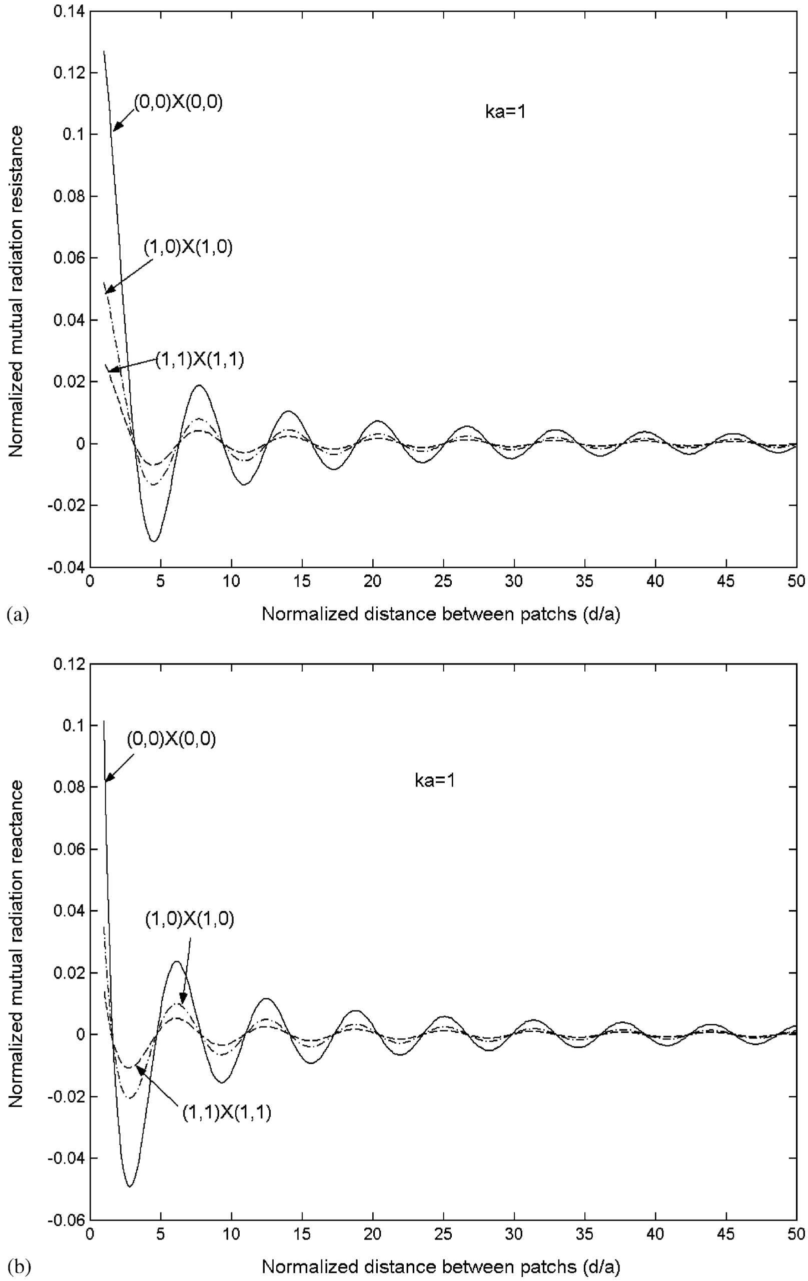


Fig. 4. (a) Normalized mutual-radiation resistance for two square patches of dimension $ka = 1$ and velocity distribution (m, n) mode $\times$ (k, l) mode as a function of the normalized separation distance d/a. (b) Normalized mutual-radiation reactance for two square patches of dimension $ka = 1$ and velocity distribution (m, n) mode $\times$ (k, l) mode as a function of the normalized separation distance d/a.

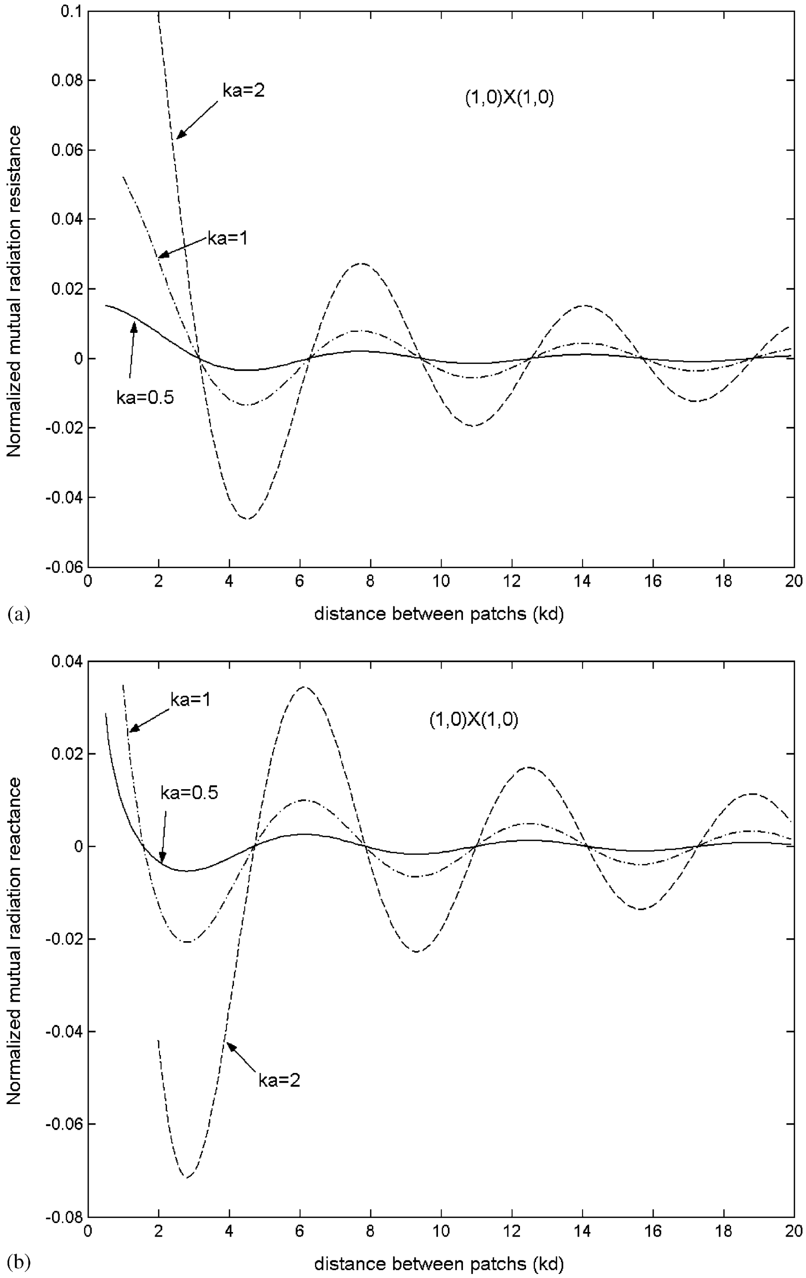
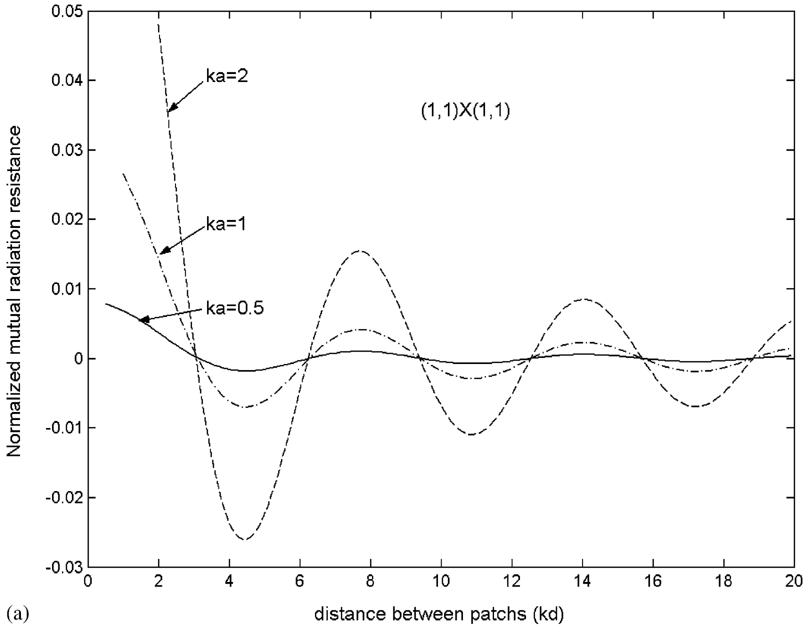
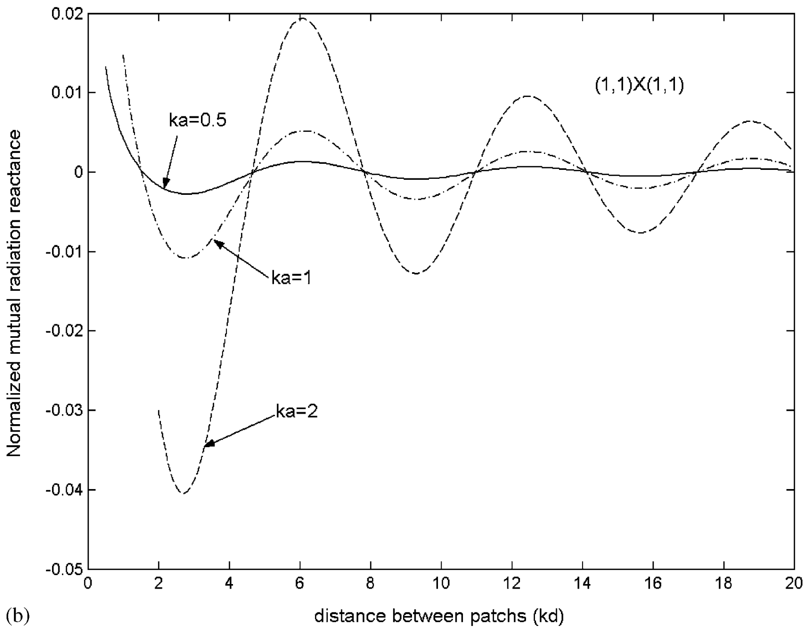


Fig. 5. (a) Normalized mutual radiation resistance for two square patches of dimension ka and velocity distribution $(1, 0)$ mode $\times(1, 0)$ mode as a function of the separation distance kd. (b) Normalized mutual radiation reactance for two square patches of dimension ka and velocity distribution $(1, 0)$ mode $\times$ $(1, 0)$ mode as a function of the separation distance kd.



(a)



(b)

Fig. 6. (a) Normalized mutual radiation resistance for two square patches of dimension ka and velocity distribution (1, 1) mode $\times$(1, 1) mode as a function of the separation distance kd. (b) Normalized mutual radiation reactance for two square patches of dimension ka and velocity distribution (1, 1) mode $\times$ (1, 1) mode as a function of the separation distance kd.

showed that the computation algorithm is more efficient than the quadruple integrals by using a standard numerical quadrature algorithm. The proposed method can be extended to arbitrarily vibrating rectangular patch radiators for calculation of the self- and mutual-radiation impedance.

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